Open with

Example 15

A spear is thrown over level ground from the top of a tower.
The height, in metres, of the spear above the ground after t seconds is modelled by the function:

$$h(t) = 12.25 + 14.7t - 4.9t^2,\ t \geqslant 0$$

a Interpret the meaning of the constant term 12.25 in the model.

b After how many seconds does the spear hit the ground?

c Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.

d Using your answer to part **c** or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached.

Set $t = 0$

$h(0) = 12.25$

$t = 0$

height above ground

before anything has happened

a The tower is 12.25 m tall, since this is the height at time 0.

b When the spear hits the ground, the height is equal to 0.

$$12.25 + 14.7t - 4.9t^2 = 0$$

Using the formula, where $a = -4.9$, $b = 14.7$ and $c = 12.25$,

$$t = \frac{-14.7 \pm \sqrt{14.7^2 - 4(-4.9)(12.25)}}{(2 \times -4.9)}$$

$$t = \frac{-14.7 \pm \sqrt{456.19}}{-9.8}$$

$t = -0.679$ or $t = 3.68$ (to 3 s.f.)

As $t \geqslant 0$, $t = 3.68$ seconds (to 3 s.f.).

c $12.25 + 14.7t - 4.9t^2$

$= -4.9(t^2 - 3t) + 12.25$

$= -4.9((t - 1.5)^2 - 2.25) + 12.25$

$= -4.9((t - 1.5)^2 + 11.025 + 12.25)$

$= 23.275 - 4.9(t - 1.5)^2$

So $A = 23.275$, $B = 4.9$ and $C = 1.5$.

d The maximum height of the spear is 23.275 metres, 1.5 seconds after the spear is thrown.

Problem-solving

Read the question carefully to work out the meaning of the constant term in the **context of the model**. Here, $t = 0$ is the time the spear is thrown.

To solve a quadratic, factorise, use the quadratic formula, or complete the square.

Give any non-exact numerical answers correct to 3 significant figures unless specified otherwise.

Always interpret your answers in the context of the model. t is the time after the spear was thrown so it must be positive.

$4.9(t - 1.5)^2$ must be positive or 0, so $h(t) \leqslant 23.275$ for all possible values of t.

The turning point of the graph of this function would be at (1.5, 23.275). You may find it helpful to draw a sketch of the function when working through modelling questions.

BitPaper
Share Knowledge

$2x+2y=7$ ① $x=\frac{7-2y}{2}$

$x^2-4y^2=8$ ②

$\left(\frac{7-2y}{2}\right)^2-4y^2=8$

$\left(\frac{7-2y}{2}\right)\left(\frac{7-2y}{2}\right)-4y^2=8$

$\frac{49-14y-14y+4y^2}{4}\times 4 \quad -4y^2=8 \quad \times 4 \quad \times 4$

$49-28y+4y^2-16y^2=32$

$-12y^2-28y+17=0$

$12y^2+28y-17=0$

$y=\frac{-28\pm\sqrt{28^2-4(12)(-17)}}{2\times 12}$

$y=\frac{1}{2}$, $y=\frac{-17}{6}$

$x=\frac{7-2y}{2}$, $x=\frac{7-2y}{2}$

$x=3$, $x=19/3$

BitPaper
Share Knowledge

$3x - y\ 2$ ① $\Rightarrow y = 3x\ 2$

$x^2 + 5ky + 3k - 0$ ②

$x^2 + 5 = (3x\ 2) + 3k = 0$

$x^2 + 15tx\ 10k + 3k = 0$

$x^2 + 15kx - 7k = 0$

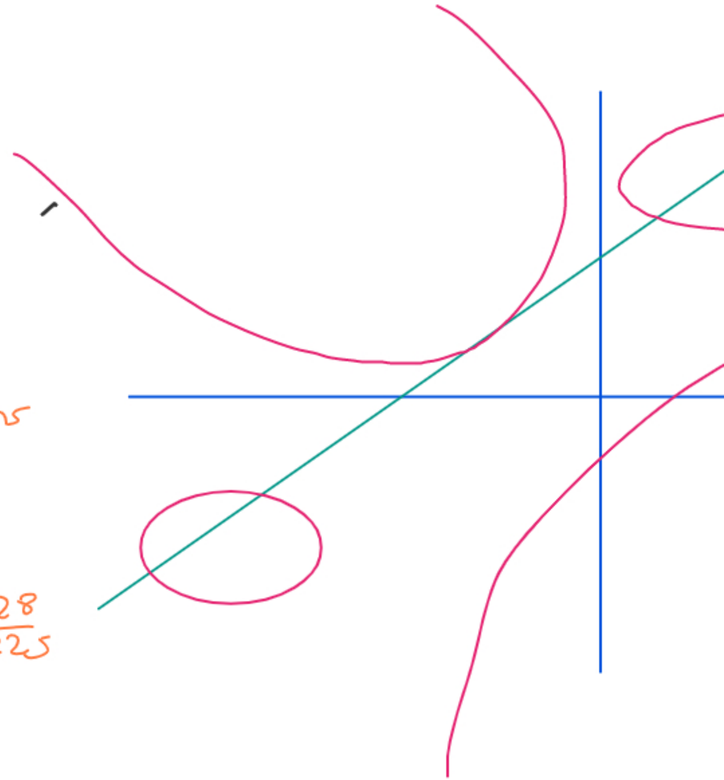
$a = 1 \quad b = 15k \quad c = 7k$

$D = b^2 - 4ac$

$= (5k)^2 - 4(1)(-7k)$

$= 225k^2 + 28k$

$D = b^2 - 4ac > 0$ or $= 0$ < 0



a) no intersections

$D = b^2 - 4ac < 0$

$225k^2 + 28k < 0$

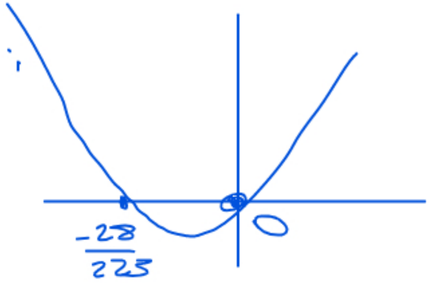
$k(225k + 28) < 0$

critical values:

$k = 0 \qquad 225k + 28 = 0$

$k = \frac{-28}{225}$

$\frac{-28}{225} < k < 0$



b) 1 intersection

$b^2 - 4ac = 0$

$k = 0 \qquad k = -\frac{28}{225}$

c) 2 intersections

$b^2 - 4ac > 0$

$k > 0$ or $k < -\frac{28}{225}$

$y = 3x - 4$

$kx^2 - 2xy + 5k = 0$ given $k \neq 0$

Find the values of k such that the line and curve

a) Intersect twice

b) Intersect once

c) Do not intersect

$kx^2 - 6x^2$

$(k - 6)x^2$

Solution is a) $-0.49 < k < 6.49$

b) $k = -0.49$ $k = 6.49$

c) $k < -0.49$ or $k > 6.49$

BitPaper
Share Knowledge

Homework

Send me the correct working for the question on the last slide

Questions 7, 8 and 9 from Ex 3C (page 45) email me the solutions to those.
Deadline is Tuesday 9pm

$y = 3x - 4$

$kx^2 - 2xy + 5k = 0$ given $k \neq 0$

Show that

Find the values of k such that the line and curve

a) Intersect twice

b) Intersect once

c) Do not intersect

a) $-0.49 < k < 6.49$

b) $k = -0.49$ $k = 6.49$

c) $k < -0.49$ or $k > 6.49$