GCSE Mathematics: Percentages

Priority Learning

Revision Sheet

Name: ______________________________ **Date**: ______________

Aims of this worksheet:

- Understanding basic percentages.
- Understanding percentage increase and decreases.
- Understanding reverse percentages.

Key Facts To Remember

- To convert a percentage into a decimal we divide by 100.
- Read the question very carefully. Think about what the question is asking for.

Basic Percentages

To find the percentage of a number we multiply the number by the percentage expressed as a decimal.

Example

Find 14% of 56.

Solution:

$$14\% = 0.14$$
$$0.14 \times 56 = 7.84$$

Example

Find 82% of 92.

Solution:

$$82\% = 0.82$$
$$0.82 \times 92 = 75.44$$

Percentage Increase/Decrease

- To find a percentage increase:
  - We add the % increase to 100%.
  - Divide by 100.
  - Multiply by the number.
- To find a percentage decrease:
  - We subtract the % decrease from 100%.
  - Divide by 100.
  - Multiply by the number.

The Percentage Formula

This is given by the formula:

$$\text{Initial value} \times \frac{100 \pm \%}{100} = \text{Final value}$$

- The most common percentage decrease questions will be items in a shop sale. You will be given the full price and asked to find the sale price.

- The most common percentage increase questions will be:
  - A person puts money into a bank or makes an investment. For these questions you will be asked to find the amount of money they have after a certain amount of time.
  - VAT on an item, VAT means value added tax, which is a set % added on to the price of the item. These questions will usually give the price of the item without VAT and ask you to add it on.
  - A service charge on a food bill. These questions will usually give you the price of a food bill and ask you to add on a % service charge.

Example

A shirt costs £20. The shop then puts a 15% off sale on. What is the sale price?

Solution:

$$\begin{aligned} 100\% - 15\% &= 85\% \\ 85 \div 100 &= 0.85 \\ 0.85 \times £20 &= £17. \end{aligned}$$

Example

James puts £2000 into his savings account. The interest is 2.5% per year. How much money will James have in 1 years time?

Solution:

$$100\% + 2.5\% = 102.5\%$$
$$102.5\% \div 100 = 1.025$$
$$1.025 \times £2000 = £2050$$

Reverse Percentages - Method

Reverse percentages are a little different. To understand them you need to be completely confident with the previous sections and translating information into algebra.

- When we figure out a normal percentage we multiply our "original number" - the number before any percentage has been applied to it - by our percentage (converted into a decimal).

- In a reverse percentage problem we don't know what the original number was, but we are given the percentage and the number after the percentage has been applied.

- We call the original price x.

The Percentage Formula

We can use the percentage formula seen earlier, this time we just have to find the initial value, as we are given the final value:

$$\text{Initial value} \times \frac{100 \pm \%}{100} = \text{Final value}$$

- You can have reverse percentage increase and decrease problems for example:
  - You may be given the sale price of an item, the % of the sale and asked to find the 'before sale' price of the item.

  - You may be given the price of a food bill including VAT. You will be asked to calculate the price of the bill before VAT was added.

Reverse Percentages - Examples

Example

A shirt in a sale costs £24. The shop has a 25% off sale on. What was the price of the shirt before the sale?

Solution:

$$\begin{aligned}
&\text{Let the full price be } x.\\
&x \text{ was decreased by } 25\%\\
&100\% \text{ -}25\% = 75\% = 0.75\\
&\text{So we have:}\\
&0.75x = £24\\
&x = £24 \div 0.75\\
&x = £32
\end{aligned}$$

Example

A food bill at a restaurant is £80 including a service charge at 20%. The restaurant says if you can figure out what the bill was without the service charge, then you don't have to pay it. What was the price of the bill before the service charge was added?

Solution:

$$\begin{aligned} &\text{Let the full price be } x. \\ &x \text{ was increased by } 20\% \\ &100\% + 20\% = 120\% = 1.2 \\ &\text{So we have:} \\ &1.2x = £80 \\ &x = £80 \div 1.2 \\ &x = £66.67 \end{aligned}$$

Example

James puts some money into his savings account. The interest is 2.5% per year. After a year James has £2050. How much money did James put in?

Solution:

Let x be the amount of money James put in.

$$\begin{aligned} 100\% + 2.5\% &= 102.5\% \\ 102.5\% \div 100 &= 1.025 \\ 1.025x &= 2050 \\ x &= \frac{2050}{1.025} \\ x &= £2000 \end{aligned}$$

Example

John has 3 dogs, he decides he wants more and adopts 6 more from a rescue centre. By what percentage did the amount of dogs John owns increase?

Solution:

John's initial amount of dogs was 3, his final amount is 9. Let the percentage increase be x.

$$\begin{aligned} 3x &= 9 \\ x &= \frac{9}{3} \\ x &= 3 \qquad\qquad = 300\% \end{aligned}$$

The difference between 100% and 300% is 200%. Hence John's dogs increased by 200%.

Compound Interest

Compound interest occurs when we apply a percentage increase or decrease a number of times.

- Add or subtract (as appropriate) the % increase or decrease to 100% as normal.
- Divide by 100.
- Multiply the number by your decimal to the power of 'how many times you apply the %'.
- The most common type of problem would be money gaining interest over a number of years in a bank.
- Again, we can use our standard formula, we now put the "fraction bit" to the power of n. Where n is the number of repetitions we are doing.

The Percentage Formula

This is given by the formula:

$$\text{Initial value} \times \left(\frac{100 \pm \%}{100}\right)^n = \text{Final value}$$

Example

Lucy's parents put £5000 into a child savings account when she is 13. She gets the money when she is 18. The interest is 4% per year. How much will she have at the end of the 5 years?

Solution:

$$\begin{aligned} 100\% + 4\% &= 104\% \\ &= 1.04 \end{aligned}$$

As we have a 5 year period we do,

$$£5000 \times (1.04)^5 = £6083.26.$$

Example

A bank offers a help to buy ISA (individual savings account used to save up to buy a house) at 2.5% annual interest. John has £4000 to put towards a house right now. He wants to buy a house in 3 years. When John withdraws his money to buy the house, the government will give him an additional 25% on his final balance. Given that the deposit on a house is 5% of it's total value, what is the maximum house price John could afford in 3 years?

Solution:

First we calculate how much John withdraws after the 3 years.

$$\begin{aligned} 100\% + 2.5\% &= 102.5\% \\ &= 1.025 \end{aligned}$$

It's a 3 year period hence our calculation is,

$$\pounds 4000 \times (1.025)^3 = \pounds 4307.56$$

We now apply the government increase,

$$\begin{aligned} 25\% \text{ increase} \quad \Longrightarrow \frac{100+25}{100} &= 1.25 \\ \pounds 4307.56 \times 1.25 &= \pounds 5384.45 \end{aligned}$$

This money will cover the deposit of the house, call the house price x. We know 5% of this price x will equal our $\pounds 5384.45$.

$$\begin{aligned} 5\% \text{ of a number} \quad \Longrightarrow \frac{5}{100} &= 0.05 \\ 0.05x &= \pounds 5384.45 \\ x &= \pounds 107,689.00 \end{aligned}$$

Example (cont...)

John decides to buy a house for £105,000. His deposit goes towards the price of the house, he decides to pay £500 a month towards paying off his mortgage, how long does it take him in years and months?

Solution:

$$105,000 - 5384.45 = 99,615.55$$

We now divide by 500 to find out how many months it takes.

$$99,615.55 \div 500 = 199.23$$

Now we divide by 12 to get how many years it takes.

$$199.23 \div 12 = 16.6 \text{ years}$$

.6 of a year is,

$$12 \times 0.6 = 7.2$$

So on the beginning of the 7$^{\text{th}}$ month it hasn't been paid off, so it will be the 8$^{\text{th}}$ month by the time it is fully paid off. So our answer is,

16 years and 8 months.