A-Level Mathematics: Moments

Priority Learning

Revision Sheet

Name: ______________________ **Date**: __________

Aims of this worksheet:

- Learning how to resolve moments.
- Learning how to solve ladder problems.

Basic Moments

In this section we explore what moments are and how to calculate them.

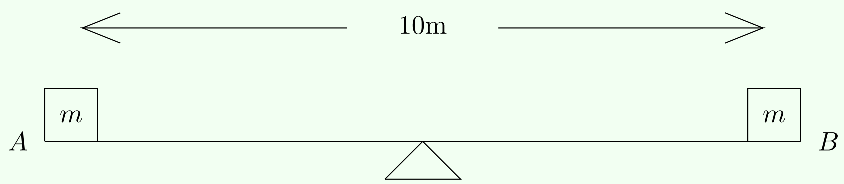
The Formula

Moment = Force × perpendicular distance from pivot.

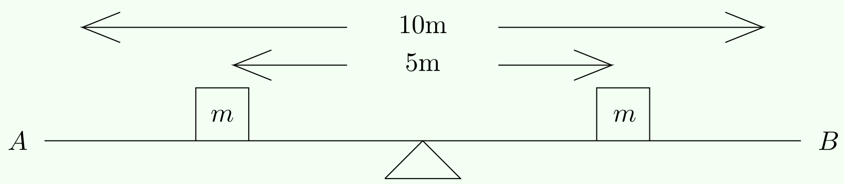
As such, the units for the moment are Nm.

Example

If two particles of the same mass (m) rest an equal distance away from the pivot then the system is in equilibrium, as shown below.



Both particles are the same mass and an equal distance from the pivot so the system is in equilibrium.



Both masses are moved an equal amount, so the rod remains in equilibrium.

Remember

For a system in equilibrium,

Clockwise moments = Anticlockwise moments

We use this fact to resolve moments problems.

Remember

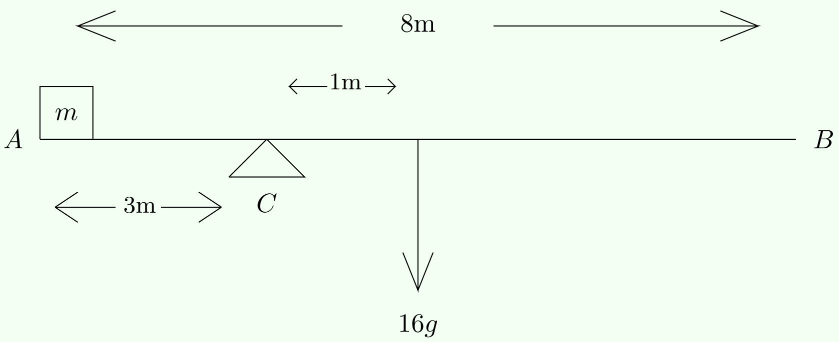
The weight of a rod acts about the centre.

Example

A uniform rod AB, of length 8m and mass 16kg rests on a pivot at the point C, 3m away from A. A particle P is placed on the end A. The system is in equilibrium, what is the mass of P.

Solution:

Let the mass of P be m.



Taking moments about C we obtain:

$$mg \times 3 = 16g \times 1$$
$$3mg = 16g$$
$$m = \frac{16g}{3}$$
$$m = 52\text{kg}$$

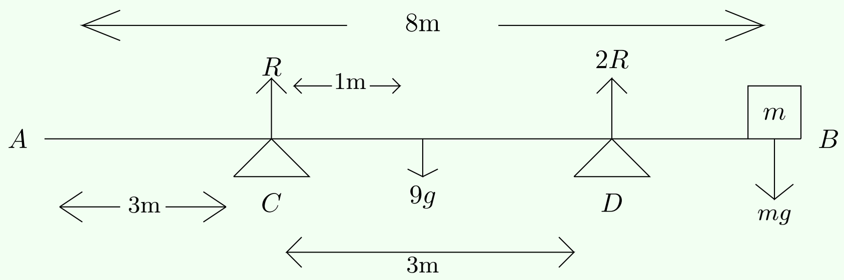
It is useful to take moments about a pivot point so we don't have to take into account the resultant force at the pivot. However, if we have 2 pivot points we will have to take one of them into account.

Example

A uniform rod AB, of length 8m and mass 9kg rests on pivots at the point C and D, with C 3m away from A and D 6m away from A. A particle P is placed on the end A. The reaction force at C is twice the reaction force at D. The system is in equilibrium, what is the mass of P.

Solution:

Let the mass of P be m.



First we need to resolve forces vertically to calculate the value of R.

$$
\begin{aligned}
9g + mg &= 2R + R \\
9g + mg &= 3R \\
R &= \frac{1}{3}g(9+m)
\end{aligned}
$$

Taking moments about C we obtain:

$$
\begin{aligned}
9g + 5mg &= 3 \times 2R \\
9g + 5mg &= 2g(9+m) \\
9 + 5m &= 18 + 2m \\
3m &= 9 \\
m &= 3 \text{ kg}
\end{aligned}
$$

Ladder Problems

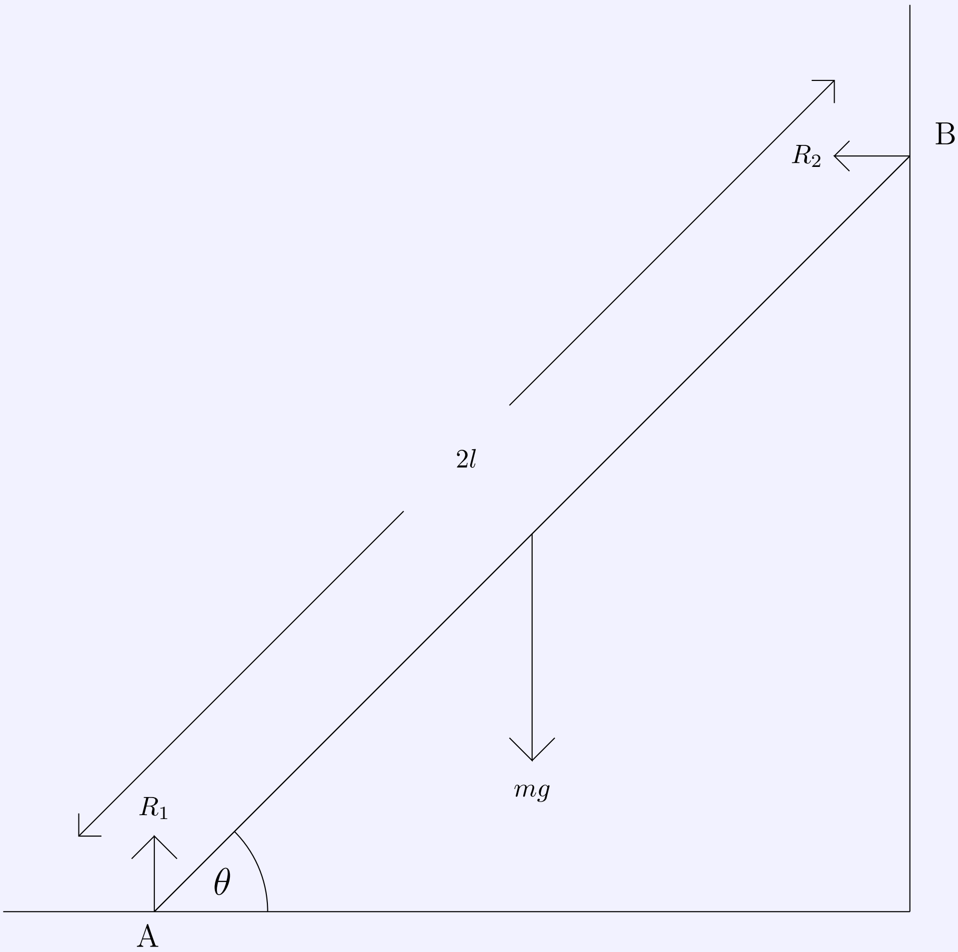
In this section we explore examples of ladder problems and how to solve them.

Remember

Moment = Force × Perpendicular Distance From Pivot

Sin or Cos?

Imagine we are taking moments about the point A in the following diagram.



The forces causing the moments about A are mg and R_2. The force mg causes a clockwise moment, the force R_2 causes a clockwise moment. So we have sorted out our forces and the direction of rotation they produce. Now we just need to decide what the 'perpendicular distance from the pivot' is.

First let's look at the force mg, it acts downwards. If we have a force acting downwards directly above A then it would produce no turning effect. So it's the horizontal distance from A that causes the force of mg to turn the ladder. The horizontal distance is $l\cos(\theta)$. So for mg we have,

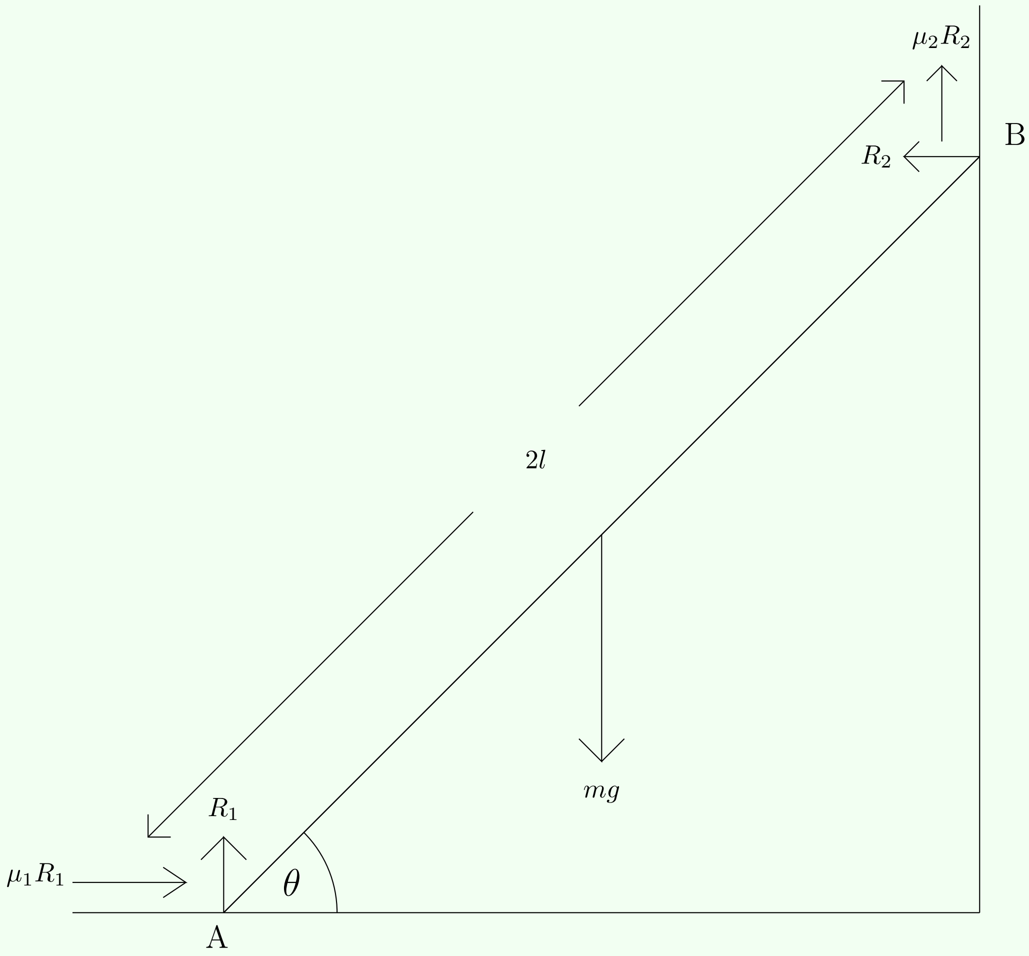
$$\begin{aligned} \text{Moment} &= \text{force} \times \text{perpendicular distance from the pivot} \\ &= mg \times l\cos(\theta) \\ &= mgl\cos(\theta) \end{aligned}$$

Similarly for the force R_2, if we had a horizontal force acting directly to the left or right of A it would not cause the ladder to rotate. But, if we have vertical force above A it will cause the ladder to rotate. So it's the vertical distance from A that causes the force of R_2 to turn the ladder. The horizontal distance is $2l\sin(\theta)$. So for R_2 we have,

$$\begin{aligned} \text{Moment} &= \text{force} \times \text{perpendicular distance from the pivot} \\ &= R_2 \times 2l\sin(\theta) \\ &= 2lR_2\sin(\theta) \end{aligned}$$

Example

Suppose a ladder AB of mass m and length $2l$ lies on a rough surface against a rough wall at an angle θ to the horizontal. The ground has coefficient of friction μ_1 and the wall has coefficient of friction μ_2. The force diagram is shown below.



If we take moments about A we obtain:

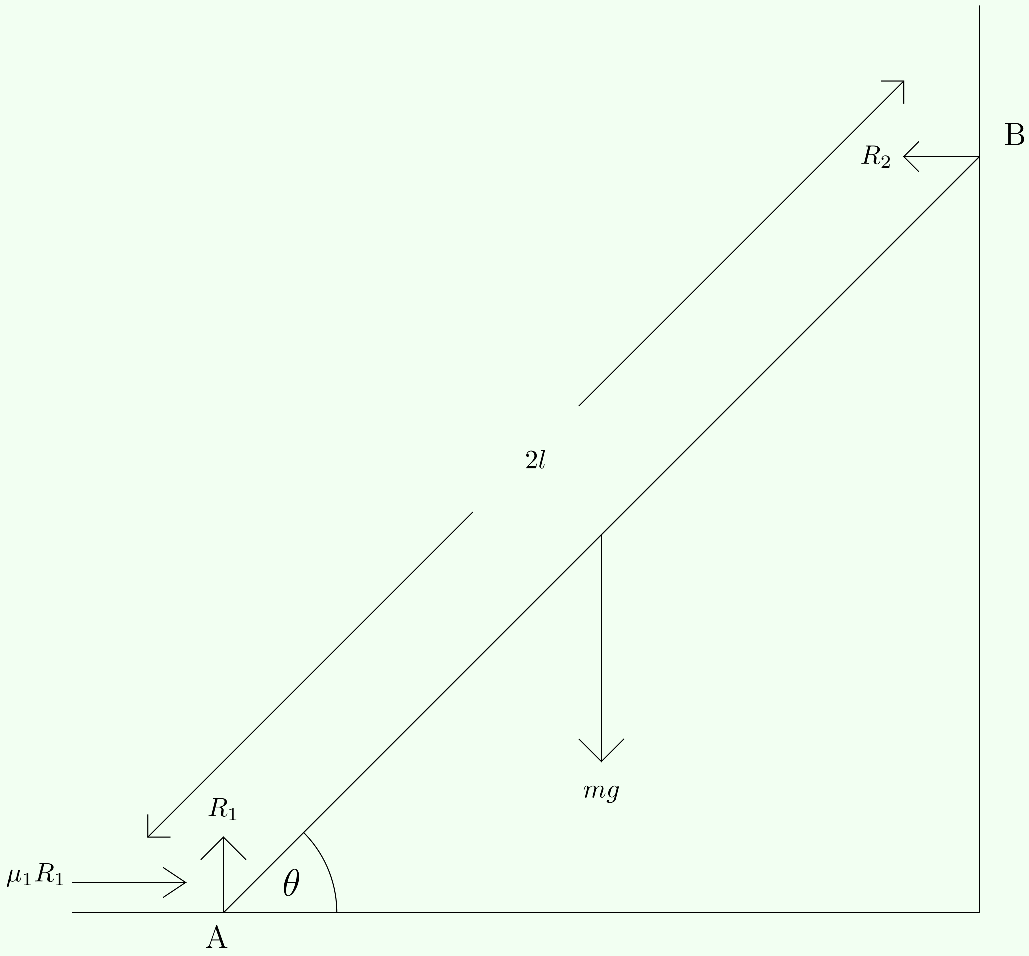
$$mgl\cos(\theta) = 2\mu_2 R_2 l\cos(\theta) + 2R_2 l\sin(\theta)$$

Because the horizontal distance from mg to A is $l\cos(\theta)$. The vertical distance does not matter. Imagine a force acting downwards directly above A. This would have no turning effect about A, so we are only concerned with the horizontal distance of mg and $\mu_2 R_2$ and conversely the vertical distance of R_2.

Example

Suppose a ladder AB of mass m and length $2l$ lies on a rough surface against a smooth wall at the point B. The ladder is at an angle θ to the horizontal. The ground has coefficient of friction μ_1. Given that the normal reaction at B is half the normal reaction at A and that the ladder is in equilibrium. Show that $\tan(\theta) = 1$

Solution:



If we take moments about A we obtain:

$$mgl\cos(\theta) = 2R_2 l\sin(\theta)$$
$$mg\cos(\theta) = 2R_2\sin(\theta)$$
$$m = \frac{2R_2\sin(\theta)}{g\cos(\theta)}$$

Example (cont...)

Resolving vertically we have,

$$mg = R_1$$

We know that,

$$\begin{aligned} \frac{1}{2}R_1 &= R_2 \\ R_1 &= 2R_2 \end{aligned}$$

Hence,

$$\begin{aligned} mg &= 2R_2 \\ R_2 &= \frac{1}{2}mg \end{aligned}$$

So our equation becomes,

$$m = \frac{2 \times \frac{1}{2}mg\sin(\theta)}{g\cos(\theta)}$$

The g terms cancel and the $\sin(\theta)$ and $\cos(\theta)$ divided make $\tan(\theta)$. So we have,

$$\begin{aligned} m &= m\tan(\theta) \\ \tan(\theta) &= 1 \\ \theta &= 45 \end{aligned}$$