A-Level Mathematics: Kinematics

Priority Learning

Revision Sheet

Name: ______________________________ **Date**: ______________

Aims of this worksheet:

- Learning the suvat equations.
- Learning how to translate a question into a suvat.
- Practice common exam style questions.

The Equations

This section covers the suvat equations and what the variables mean.

The Variables

- s = Displacement
- u = Initial velocity
- v = Final velocity
- a = Acceleration
- t = Time

Remember: for a falling object the acceleration is 9.8ms^{-1}.

Velocity

Velocity is displacement per unit time.

Acceleration

Acceleration is the change in velocity per unit time. Change in velocity is $v - u$.

Which leads us to the following equations.

The Equations

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$a = \frac{v - u}{t}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

Example

A ball is thrown up in the air with velocity 10ms^{-1}. How long does it take for it to hit the ground?

Solution: To solve any kinematics question we just write down the variables we know and what we want to find out.

$$u = 10$$
$$v = -10$$

Because the velocity just before the ball hits the ground has the same magnitude as the velocity when the ball is thrown, but the direction is opposite hence the minus sign.

$$s = 0$$

Because the displacement is zero. The ball is thrown and lands in the same place.

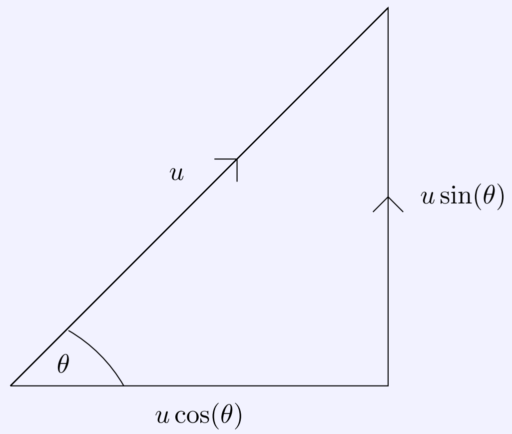
$$a = -9.8$$
$$t = ?$$

The simplest equation to use here would be,

$$\begin{aligned} v &= u + at \\ 10 &= -10 + 9.8t \\ 20 &= 9.8t \\ t &= 2.04s \end{aligned}$$

Motion For A Particle Travelling At An Angle

When a particle is travelling at an angle is has velocity in both the vertical and horizontal directions.



Example

A ball is thrown at an angle 30 degrees to the horizontal with initial velocity 8ms^{-1}. How far away does it land?

Solution: We resolve kinematics horizontally and vertically.

Vertically we have,

$$\begin{aligned} u &= 8\sin(30) = 4 \\ v &= -8\sin(30) = 4 \\ s &= 0 \\ a &= -9.8 \\ t &= t = ? \end{aligned}$$

Horizontally we have,

$$\begin{aligned} u &= 8\cos(30) = 6.93 \\ v &= 8\cos(30) = 6.93 \\ a &= 0 \\ s &= ? \\ t &= t = ? \end{aligned}$$

If we resolve horizontally we won't get very far. The vertical displacement is zero so that doesn't help us find the horizontal displacement directly. So the best way to solve this is find the time using the vertical motion then use that to find the horizontal distance. So vertically we have,

$$\begin{aligned} v &= u + at \\ -8 &= 8 - 9.8t \\ -16 &= -9.8t \\ t &= 1.63s \end{aligned}$$

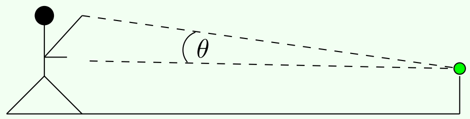
We can now use this horizontally to calculate the distance. As the acceleration is zero we can use,

$$\begin{aligned} v &= \frac{s}{t} \\ s &= tv \\ s &= 1.63 \times 6.93 \\ s &= 11.3\text{m} \end{aligned}$$

Most questions that involve a particle propelled at an angle and require a horizontal distance to be calculated are solved in a similar fashion to this one.

Example

A tennis ball is served from a height of 2m above the ground. The player hits the ball at an angle θ as shown in the diagram. The magnitude of the velocity of the serve is 10ms^{-1} and the net is 12m away and 0.8m tall. What is the minimum angle the tennis player needs to serve at in order for the ball to clear the net?



Solution:

Note that the vertical displacement must be at least the height of the net. So vertically we have,

$$
\begin{aligned}
u &= 10\sin(\theta) \\
v &= ? \\
a &= 9.8 \\
s &> 2 - 0.8 = 1.2 \\
t &= ?
\end{aligned}
$$

Horizontally we have,

$$
\begin{aligned}
u &= 10\cos(\theta) \\
v &= 10\cos(\theta) \\
a &= 0 \\
s &= 15 \\
t &= ?
\end{aligned}
$$

First we must calculate an expression time taken for the ball to reach the net, because time is irrespective of vertical or horizontal components. Horizontally we have,

$$
\begin{aligned}
v &= \frac{s}{t} \\
\implies t &= \frac{s}{v} \\
t &= \frac{15}{10\cos(\theta)} \\
t &= \frac{3}{2\cos(\theta)}
\end{aligned}
$$

If we can form an equation from the vertical components with θ and t the only variables then we have simultaneous equations we can solve.

$$
\begin{aligned}
s &= ut + \frac{1}{2}at^2 \\
1.2 &= 10t\sin(\theta) + 4.9t^2
\end{aligned}
$$

We substitute our $t = \dfrac{3}{2\cos(\theta)}$ into our vertical equation.

Example (cont..)

Solution:

$$1.2 = 10\sin(\theta)\left(\frac{3}{2\cos(\theta)}\right) + 4.9\left(\frac{3}{2\cos(\theta)}\right)^2$$

$$1.2 = 15\tan(\theta) + 22.05\sec^2(\theta)$$

We now use the identity,

$$\tan^2\theta + 1 = \sec^2(\theta)$$

$$1.2 = 15\tan(\theta) + 22.05(\tan^2\theta + 1)$$

$$15\tan(\theta) + 22.05\tan^2(\theta) + 22.05 - 1.2 = 0$$

$$22.05\tan^2(\theta) + 15\tan(\theta) + 20.85 = 0$$

Using the quadratic formula we have:

$$\tan(\theta) = \frac{-15 \pm \sqrt{15^2 - 4 \times 22.05 \times 20.85}}{2 \times 22.05}$$

$$\tan(\theta) = 1.634 \text{ or } \tan(\theta) = 2.314$$

We use the positive solution hence,

$$\theta = \tan^{-1}(1.634)$$

$$\theta = 58.53$$

Example

Two cars are in a race, a blue car and a red car. The red car is 200m ahead of the blue car. The red car is travelling at a constant velocity of 12ms^{-1}. The blue car is travelling at 5ms^{-1} and then begins to accelerate at 1.5ms^{-2}. It is a straight track and the finish line is 250m away from the red car. Who will win the race?

Solution: Say that the blue car overtakes the red car at some point X. Which is a distance d away from the red car and hence $d + 200$ away from the blue car.

$u = 5$	$u = 12$
$a = 1.5$	$a = 0$
$t = t$	$t = t$
$s = d + 200$	$s = d$
$v = ?$	$v = 12$

$\longleftarrow d \longrightarrow$

From these 2 sets of variables we see d and t are common to both particles. So we desire to form an equation including t and d for both the red and blue cars. For the red car we have,

$$v = \frac{s}{t} \text{ as the acceleration is zero}$$
$$12 = \frac{d}{t}$$

For the blue car,

$$s = ut + \frac{1}{2}at^2$$
$$d + 200 = 5t + 0.75t^2$$
$$d = 0.75t^2 + 5t - 200$$

Substituting this into our equation for the red car we obtain,

$$12 = \frac{0.75t^2 + 5t - 200}{t}$$
$$12t = 0.75t^2 + 5t - 200$$
$$0.75t^2 - 7t - 200 = 0$$

Using the quadratic formula and taking the positive solution we obtain,

$$t = 21.65$$

Substituting this into our red car equation we have,

$$12 = \frac{d}{21.65}$$
$$d = 259.8\text{m}$$

So the blue car would only overtake the red car after 259.8m which is after the finish line. So the red car wins.