A-Level Mathematics: Dynamics

Priority Learning

Worksheet

Name: ______________________________ **Date**: ______________

Aims of this worksheet:

- Learning how to resolve forces for moving systems.
- Extending physics knowledge learned from statics.

Key Facts

When we solve dynamics questions we need to use the following facts.

Key Facts

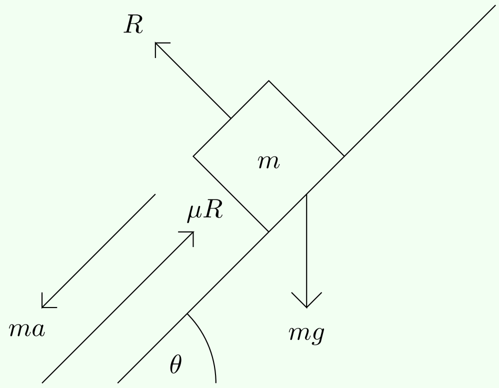
- The acceleration of connected particles is the same.
- The tension in the same part of a string or spring, is the same.
- Resolve forces particle by particle in 2 particle problems.
- Use F = ma where F = the equation for the net force on the system, such that the forces that are positive act in the same direction as the acceleration. Conversely, the forces that act in the opposite direction to the acceleration are negative as they oppose the acceleration.
- We always start with $ma =$ ... then resolve forces on the right hand side of that equation. Strictly speaking ma equals the net sum of the forces acting on the particle.
- So our equation would be:

  – $ma = \Sigma$ (F same direction as a) - Σ (F opposite direction as a).

Example

A particle of mass m lies on a rough slope inclined at an angle θ to the horizontal. The coefficient of friction between the slope and particle is μ. The particle is released from rest. Find an expression for the acceleration of the particle.

Solution:



We resolve forces parallel to the plane and perpendicular to the plane. This is because ma acts parallel to the plane so we want to resolve forces in the same direction as ma in order to make the problem as simple as possible.

Trigonometry Reminder

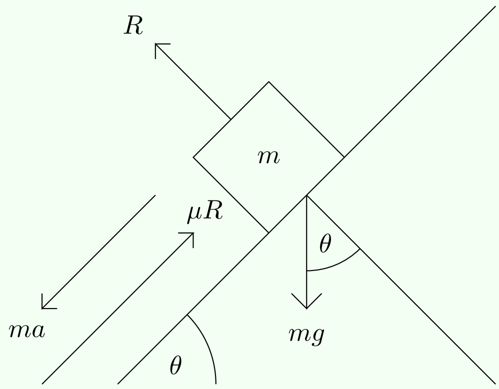
The component of mg acting perpendicular to the plane is $mg\sin(\theta)$. This is because mg is opposite to the angle θ so by sohcahtoa we use $\sin(\theta)$. We multiply $\sin(\theta)$ by mg as this gives us the 'part' of mg that acts parallel to the slope.

Parallel to the plane:

$$ma = mg\sin(\theta) - \mu R$$

Perpendicular to the plane:

If we extend the line of R down then use the knowledge we learned in GCSE angles we obtain the following triangle.



Hence, the component of mg perpendicular to the plane is $mg\cos(\theta)$.

Example (cont...)

Hence,

$$R = mg\cos(\theta)$$

The parallel and perpendicular resolution of forces equations give us 2 simultaneous equations, which we can solve for a.We now substitute $R = mg\cos(\theta)$ into our other equation.

$$ma = mg\sin(\theta) - \mu mg\cos(\theta)$$

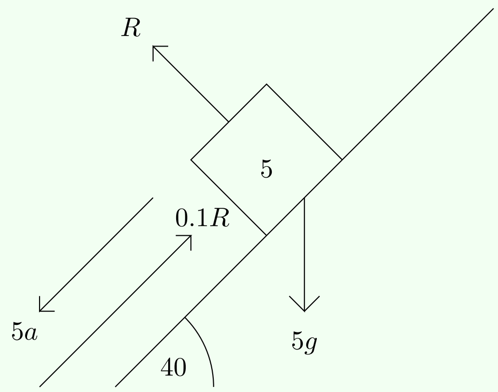
Hence,

$$a = \frac{mg\sin(\theta) - \mu mg\cos(\theta)}{m}$$
$$a = g\sin(\theta) - \mu g\cos(\theta)$$

Example

A particle of mass 5kg lies on a rough slope inclined at an angle 40 degrees to the horizontal. The coefficient of friction between the slope and particle is 0.1. The particle is released from rest. Find an expression for the acceleration of the particle.

Solution:



Parallel to the plane:

$$5a = 5g\sin(40) - 0.1R$$

Perpendicular to the plane:

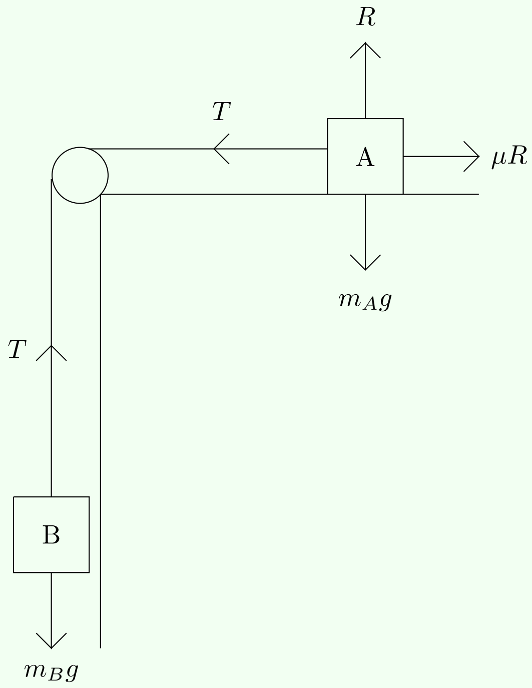
Hence,

$$R = 5g\cos(40)$$
$$5a = 5g\sin(40) - 0.1 \times 5g\cos(40)$$
$$5a = 27.743$$
$$a = 5.55ms^{-2}$$

Example

Two particles A and B of mass 6kg and 4kg respectively. Particle A rests on a rough horizontal table (shown below) with coefficient of friction 0.4. Find the acceleration of the particles in terms of g.

Solution: We solve the system on a particle by particle basis.



Particle A:

$$
\begin{aligned}
6a &= T - 0.4R \\
R &= 6g \\
6a &= T - 2.4g
\end{aligned}
$$

Particle B:

$$4a = 4g - T$$

We now solve as simultaneous equations.

$$
\begin{aligned}
T &= 6 + 2.4g \\
4a &= 4g - 6a - 2.4g \\
10a &= 1.6g \\
a &= 0.16g \text{ ms}^{-2}
\end{aligned}
$$

Example (cont...)

We attach masses to A in order to put the system in equilibrium. What mass would we need to add to A need to be in order to put the system in equilibrium?

Solution:

We solve the system on a particle by particle basis again. Let the total mass of A be m.

Particle A:

$$ma = T - 0.4R$$
$$R = mg$$
$$ma = T - 0.4mg$$

Particle B

$$4a = 4g - T$$

We now solve as simultaneous equations.

$$T = ma + 0.4mg$$
$$4a = 4gma - 0.4mg$$
$$4a - ma = 4g - 0.4mg$$

The system is in equilibrium so a=0.

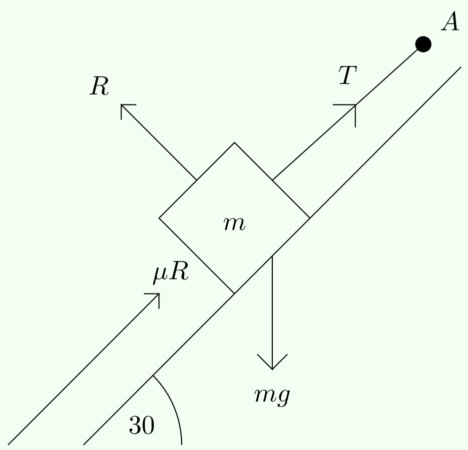
$$0 = 4g - 0.4mg$$
$$0.4m = 4$$
$$m = 10kg$$

So we need to add 4kg to A.

Example

A particle of mass m is attached to a fixed point A by an elastic string. The particle is on a rough plane, with coefficient of friction μ inclined at 30 degrees to the horizontal as shown below. The tension in the string is T. What is the acceleration of the particle in terms of m, g, T and μ?

Solution:



Resolving parallel to the plane we have:

$$ma = mg\sin(30) - \mu R - T$$

As μR and T oppose the acceleration and the weight of the particle causes it.

Solving perpendicular to the plane gives us:

$$R = mg\cos(30)$$

Substituting into the other equation:

$$ma = mg\sin(30) - \mu mg\cos(30) - T$$
$$a = \frac{mg\sin(30) - \mu mg\cos(30) - T}{m}$$